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Satbayev University

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Satbayev University

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QUANTITATIVE ESTIMATES OF THE TRANSIENT PROCESS OF THE NON-CONTACT GYROSCOPE ROTOR

Abstract. The article focuses on finding quantitative estimates of the transient process of the rotor of a non-contact gyroscope, leading to stationary rotation around the axis of the greatest moment of inertia. The Euler - Poinsot solution is used as generating the solution to the problem, and the influence of the elasticity of the rotor, in accordance with the ideas of the perturbation method, is considered as a small perturbation of the Euler motion.

The deformations of the gyroscope rotor are determined. The authors assume that the internal friction in the material complies with the Kelvin-Voigt hypothesis.

An expression for the nutation angle as a function of time is obtained, and the time constant of the process of damping of nutational oscillations of the rotor is determined.

The drifts are found for a real gyroscope, taking into account the aspherization of the rotor.

Key words: non-contact gyroscope, Euler angles, forces of inertia, Legendre polynomial, Poisson's ratio, dissipative function, internal friction, equation of the gyroscope nutation angle.

Introduction. The article investigates disturbing moments acting on the electrostatic (ESG) rotor, which makes the Euler-Poinseau movement. The ESG rotor is a spherical shell placed inside an evacuated jacket. A system of electrodes is located on the inner surface of the jacket, with the help of which an electrostatic field is created, providing non-contact suspension of the rotor.

The gyroscope rotor is preliminarily untwisted by the subsystem and, in the operating mode, performs the Euler-Poinsot movement. In an electrostatic suspension, the supporting forces act along the normal to the rotor surface, therefore, if its surface is an ideal sphere, then the moment of ponderomotive forces relative to the center of the sphere is zero, and if the center of the rotor mass coincides with its geometric center, the kinetic moment of the gyroscope will keep the same direction in space. In actuality, the surface of the rotor is not ideally spherical; therefore, in a real electrostatic suspension, the disturbing moment arises, caused by the non-sphericity of the rotor, which largely determines the accuracy of the instrument.

The double rotation mode of the ESG rotor is given to get rid of the disturbing moments caused by odd harmonics in the shape of its surface. However, during the Euler-Poinsot movement, the inertial forces acting on the rotor cause shape deformation of its surface along even harmonics. The influence of these harmonics on the accuracy of the instrument is analyzed. A gyroscope is considered, the rotor of which has the form of a spherical elastic shell of radius R with the same holes in the poles.

The Euler-Poinsot movement is taken as the generating solution of the problem, and the influence of the elasticity of the rotor, in accordance with the ideas of the perturbation method, is considered as a small perturbation of the Euler movement.

Materials and methods. Transient process leading to stationary rotation of the viscoelastic rotor around the axis of the greatest moment of inertia

Let us write out the dynamic Euler equations describing the rotor movement of the electrostatic gyroscope relative to the center of mass, in the trihedron *x:*

$$\frac{dL}{dt} + \omega \cdot L = 0 \tag{1.1}$$

Here L – vector of the angular momentum of the rotor relative to the center of mass, – vector of angular velocity of the trihedron *x*.

In the case when the ellipsoid of inertia is an ellipsoid of rotation about the axis Ox_3, where fore $l_l = I_3$will be more than l_l and I_2, if the ellipsoid is compressed, and will be less than these values if it is elongated. Equation projection (1.1) on the axis Ox_3equals $I_3 \frac{d\omega_3}{dt} = 0$, therefore, $\omega_3 = b$ – the projection of the angular velocity vector onto the x_3 axis at the initial moment of time.

To determine the rotor position relative to the fixed axes $\xi_1, \xi_{2,}\xi_3$ it will be enough to know the three Euler angles ϑ, φ, ψ in time function, the projections $\omega_1, \omega_2, \omega_3$ of the vector of the instantaneous angular velocity of the rotor rotation on the movable axes Ox_1, Ox_2, Ox_3 are determined by the Euler kinematic equations:

$$\omega_1 = \acute{\varphi}\, sin\,\vartheta\, sin\,\varphi + \acute{\vartheta}\, cos\,\varphi\,;$$
$$\omega_2 = \acute{\varphi}\, sin\,\vartheta\, cos\,\varphi - \acute{\vartheta}\, sin\,\varphi;$$
$$\omega_3 = \acute{\varphi}\, cos\,\vartheta + \acute{\varphi};$$

To calculate the three Euler angles as functions of time, we assume that the constant direction η_3 of the angular momentum L, known from the initial conditions.

The projections L - the vector of the angular momentum of the rotor on the movable axes are equal:

$$I_1\omega_1 = L \sin\vartheta \sin\varphi$$
$$I_2\omega_2 = L \sin\vartheta \cos\varphi$$
$$I_3\omega_3 = L \cos\vartheta \tag{1.2}$$

From the last equation (1.2) it is seen that the ϑ angle must remain constant $\vartheta = \vartheta_0$, herewith $\cos\vartheta = bI_3/L$. Taking into account the expressions for the projection of the vector of the instantaneous angular velocity of the rotor rotation on the movable axes Ox_1, Ox_2,Ox_3 and for the axis of the symmetrical rotor ($I_1 = I_2$) we have

$$\frac{d\psi}{dt} = \frac{L}{I_1}; \qquad \frac{d\varphi}{dt} = \frac{(I_3 - I_1)}{I_1}\frac{L}{I_3} cos\,\vartheta$$

Therefore, andφchange proportionally to time:

$$\psi = \frac{L}{I_1}t; \varphi = \frac{\pi}{2} - \vartheta t \tag{1.3}$$

where

$$\vartheta = b\frac{(I_3 - I_1)}{I_1}; \qquad b = \frac{L}{I_3} cos\,\vartheta$$

Instantaneous angular speed of rotation $\overline{\omega}$ is the geometric sum of three angular velocities $\acute{\vartheta}, \acute{\varphi}, \acute{\psi}$, directed respectively along OI, $O\eta_3, Ox_3$.

In this case $\acute{\vartheta}$ equals to zero, andare constant. The locus of instantaneous axes of rotation ωin the gyroscope rotor has a circular cone with the axis Ox_3.

The locus of instantaneous axes in space is a circular cone with the axis $O\eta_3$. The movement of the gyroscope rotor is obtained as a result of uniform rolling of one cone over the other.

In the absence of external forces, the angular momentum L has the fixed direction and the constant value

$$L = [(I_1a)^2 + (I_3b)^2]^{1/2} \tag{1.4}$$

Here, $a = \frac{L}{I_1}\sin\vartheta$ – the angular velocity vector projection $\overline{\omega}$ on the axis Ox_1 at the initial moment of time.

Taking into account (1.2) and (1.3) we obtain the following equations for the projections of the angular velocity on the moving axes Ox_1, Ox_2, Ox_3

$$\omega_1 = a\cos\vartheta t; \omega_2 = a\sin\vartheta t; \omega_3 = b; \tag{1.5}$$

Movement (1.2) - (1.5) will be taken as the generating solution of the problem, and the influence of the rotor elasticity, in accordance with the ideas of the perturbation method, will be considered as a small perturbation of the Euler movement.

Density of the force of inertia $\boldsymbol{F}$ is determined by the formula

$$F = -\rho[\omega \cdot R(\omega \cdot r) + R\acute{\omega} \cdot r + w_0 + \acute{u} + 2\omega \cdot \acute{u}]$$

Here the first three terms in parentheses are transferable acceleration, the fourth term is relative acceleration, and the last is Coriolis acceleration.

We will neglect the expression for the force $\boldsymbol{F}$ small terms $\acute{\boldsymbol{u}}$ and $2\,\omega \cdot \acute{u}$, this means that we neglect the natural vibrations of an elastic body, since we assume that the natural vibration frequency is much greater than the angular velocity of rotation. Since the free movement of the body is considered, the absolute acceleration of the center of inertia of the deformable body is equal to zero, therefore, the translational acceleration of the center of mass will also be equal to zero [1,2].

Then

$$F = -\rho R\{[\acute{\omega} \cdot r] + \acute{\omega}(\omega \cdot r) - \omega^2 r\} \tag{1.6}$$

Projecting the vector (1.6) onto the movable axes Ox_1, Ox_2, Ox_3 taking into account (1.5), we have

$$F_1 = -\rho R[-(b^2 + a^2(\sin \upsilon t)^2)x_1 + 0.5a^2 x_2 \sin 2\upsilon t + a(b+\upsilon)x_3 \cos \upsilon t]$$
$$F_2 = -\rho R[0.5a^2 x_1 \sin 2\upsilon t - (b^2 + a^2 \cos \upsilon t)x_2 + a(\upsilon + b)x_3 \sin \upsilon t]$$
$$F_3 = -\rho R[a(\upsilon - b)x_1 \cos \upsilon t + a(\upsilon - b)x_2 \sin \upsilon t - a^2 x_3]$$

Here, the x_i coordinates are dimensionless, referred to the rotor radius R.

In a triangle x_1, x_2, x_3 we introduce a spherical coordinate system r, α, β(0≤r≤R, 0≤α≤, 0≤β≤2π;) with polar axis x_3. Matrix C direction cosines between coordinate system x_1, x_2, x_3 and unit vectorsspherical coordinate system has the formula

$$C = \begin{vmatrix} \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \\ \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \end{vmatrix} \tag{1.7}$$

Taking into account (1.7), we redesign (1.6) on the axis of the spherical coordinate system *r, α, β* and, as result, we obtain

$$F_r = -\rho r R[-\left(b^2 + {a^2}/{2}\right) + \left(b^2 - {a^2}/{2}\right)(\cos\alpha)^2 + \text{a}b\sin 2\alpha\cos(\upsilon t - \beta) + 0.5\,\text{a}^2(\sin\alpha)^2\cos(2\upsilon t - 2\beta)];$$
$$F_\alpha = -\rho r R[-(b^2 - {a^2}/{2}\sin\alpha\cos\alpha + a(\vartheta + b)\cos 2\alpha\cos(\vartheta t - \beta) + 0.5a^2\sin\alpha\cos\alpha\cos(2\vartheta t - 2\beta)$$
$$F_\beta = -\rho r R[-a(b+\vartheta)\cos\alpha\sin(\vartheta t - b) + 0.5a^2\sin\alpha\sin(2\vartheta t - 2\beta) \tag{1.8}$$

For real designs of non-contact gyroscopes, the ratio $(I_3 - \text{I}_1)/\text{I}_1$ does not exceed 0.1; therefore, the terms with the factor υ in (1.8) turn out to be small. If we neglect the terms with the factor υ in (1.8), then a direct calculation can make sure that the forces (1.8) are potential

$$F = -grad\pi, \quad \pi = -\frac{R^2}{3}\rho\omega^2 r^2[I - P_2(æ)]$$
$$æ = [a^* \sin\alpha\cos(\beta - \vartheta t) + b^*\cos\alpha]$$

Here$P_2(æ) = (3æ^2 - I)/2$Legendre polynomial, $a^* = a/\omega b^* = b/\omega$. (1.9)

Since the period of free elastic rotor oscillations turns out to be much less than the period of rotor rotation around the center of mass, then the displacement vector of its points $u(u_r, u_\alpha, u_\beta)$, caused by the presence of inertial forces (1.8) can be found as a solution to the quasi-stationary problem of the spatial theory of elasticity

$$\frac{2(1-\mu)}{(1-2\mu)}graddivu - rotrotu - \frac{R^2}{G}grad\pi = 0 \tag{1.10}$$
$$\sigma_{ij}n_j|_{r=1} = 0 \tag{1.11}$$

where μ – Poisson ratio, G – shear modulus, $n(n_1, n_2, n_3)$ – normal to the rotor surface, σ_{ij} –stress component. When solving (1.10), (1.11), we will neglect the change in density in the equatorial plane of the rotor and restrict ourselves to finding the displacements for a homogeneous ball.

We introduce the coordinate system z_1, z_2, z_3, the beginning of which coincides with the center of mass of the rotor, and vector $\overline{\omega}$ of the rotor angular velocity is directed along the axis z_3. In the triangle z_i, we introduce the spherical coordinates $\alpha_1 \beta_1$ with polar axis z3. Then the problem (1.10), (1.11) is reduced to determining the displacement vector $u_r, u_{\alpha_1}, u_{\beta_1}$, which takes place when a body rotates around a «fixed» axis z_3. Using the well-known results from [1,2], we obtain

$$u_r = \frac{\rho\omega^2 R^3}{3G(7+5\mu)}[(I+\mu)r^3 - (3+2\mu)R^2 r]P_2(\cos\alpha_1)$$

$$u_{\alpha_1} = \frac{\rho\omega^2 R^3}{6G(7+5\mu)}[(2+\mu)r^3 - (3+2\mu)R^2 r]\frac{\partial P_2(\cos\alpha_1)}{\partial\alpha_1} \tag{1.12}$$
$$u_{\beta_1} = 0$$

Substituting into (1.12) the connection between the spherical coordinates α, β and α_1, β_1, we obtain the final relations that give a solution to problem (1.10), (1.11)

$$u_r = \frac{\rho\omega^2 R^3}{3G(7+5\mu)}[(I+\mu)r^3 - (3+2\mu)R^2 r]P_2(æ)$$
$$u_\alpha = \frac{\rho\omega^2 R^3}{6G(7+5\mu)}[(2+\mu)r^3 - (3+2\mu)R^2 r]\frac{\partial P_{2(æ)}}{\partial\alpha}$$
$$u_\beta = \frac{\rho\omega^2 R^3}{6G(7+5\mu)}[(2+\mu)r^3 - (3+2\mu)R^2 r]\frac{I}{\sin\alpha}\frac{\partial P_{2(æ)}}{\partial\beta} \tag{1.13}$$

Using (1.9) for the Legendre polynomial P2 (æ) in (1.13), we have

$$P_2(æ) = \frac{1}{2}\{\frac{3}{\omega^2}[\frac{\alpha^2}{2} + \left(b^2 - \frac{\alpha^2}{2}\right)(\cos\alpha)^2 + ab\sin 2\alpha\cos(\beta - \vartheta t) + \frac{a^2}{2}(\sin\alpha)^2\cos(2\vartheta t - 2\beta)] - 1\} \quad (1.14)$$

Note that in formulas (1.14), the time-constant terms describing the centrally symmetric deformation of the rotor are omitted. These terms do not affect its periodic deformations and, therefore, are insignificant for us in the future.

Results. Differentiating the formula (1.13) with respect to time, we find the components of the relative speed of the points of the gyroscope rotor

$$\dot{u}_r = \frac{P}{G}\nu D(r)[ab\sin 2a\sin(\nu t - \beta) + a^2(\sin\alpha)^2\sin(2\nu t - 2\beta)]$$

$$\dot{u}_\alpha = \frac{P}{G}\nu C(r)[ab\cos 2a\sin(\nu t - \beta) + \frac{a^2}{2}\sin 2\alpha\sin(2\nu t - 2\beta)]$$

$$\dot{u}_\beta = \frac{P}{G}\nu C(r)[ab\cos\alpha\cos(\nu t - \beta) + a^2\sin\alpha\cos(2\nu t - 2\beta)] \quad (1.15)$$

where

$$D(r) = \frac{R^3}{2(7+5\mu)}[(I+\mu)r^3 - (3+2\mu)r];$$

$$C(r) = \frac{R^3}{2(7+5\mu)}[(2+\mu)r^3 - (3+2\mu)r];$$

Thus, the gyroscope rotor at the nutation angle $\vartheta \neq 0$ and $\vartheta \neq \pi/2$ is under a cyclic load that dissipates energy. To estimate the energy loss, we assume that the internal friction in the material complies with the Kelvin - Voigt hypothesis and we introduce the Rayleigh dissipation function [3,4].

The dissipative function showing internal friction must go to zero if there is no internal movement in the body, in particular, if the body performs only translational or rotational motion as a whole, that is, it must depend not on the velocity itself, but on its gradient [5]

$$\Phi = \frac{I}{2}\int[\lambda^*\dot{\varepsilon}^2 + G^*\left(\dot{\varepsilon}_{rr}^2 + \dot{\varepsilon}_{\alpha\alpha}^2 + \dot{\varepsilon}_{\beta\beta}^2\right) + 2G^*(\dot{\varepsilon}_{r\alpha}^2 + \dot{\varepsilon}_{r\beta}^2 + \dot{\varepsilon}_{\alpha\beta}^2)]dv \quad (1.16)$$

Here λ^*, G^* - viscous friction coefficients in the rotor material $\dot{\varepsilon} = \dot{\varepsilon}_{rr} + \dot{\varepsilon}_{\alpha\alpha} + \dot{\varepsilon}_{\beta\beta}$ $\dot{\varepsilon}_{rr} \dots \dot{\varepsilon}_{\alpha\beta}$, – deformation rate, integration in (1.16) is carried out over the entire rotor volume. (The coefficients λ^* G^* are further considered small in the sense that the damping time *T1* of the natural elastic oscillations of the rotor is much longer than the period of elastic oscillations of the rotor *T0*. At the same time, for the correctness of the constructions performed in the future, it will be assumed that *T* is much less than the characteristic time of movement of the rotor relative to the center of mass).

Formulas, expressing the deformation tensor in terms of the derivatives of the components of the displacement vector in spherical coordinates α, β, r, have the form [6,7]

$$\varepsilon_{rr} = \frac{\partial u_r}{R\partial r}, \varepsilon_{\alpha\alpha} = \frac{1}{Rr}\frac{\partial u_\alpha}{\partial \alpha} + \frac{u_r}{Rr},$$

$$\varepsilon_{\beta\beta} = \frac{1}{Rr\sin\alpha}\frac{\partial u_\beta}{\partial\beta} + \frac{u_\alpha}{Rr}tg\alpha + \frac{u_r}{Rr},$$

$$\varepsilon_{r\alpha} = \frac{\partial u_r}{R\partial r} + \frac{1}{Rr}\frac{\partial u_r}{\partial\alpha} - \frac{u_r}{Rr},$$

$$\varepsilon_{\alpha\beta} = \frac{1}{Rr}\left(\frac{\partial u_\beta}{R\partial r} + \frac{1}{\sin\alpha}\frac{\partial u_\alpha}{\partial\beta} - u_\beta ctg\alpha\right),$$

$$\varepsilon_{\beta r} = \frac{1}{Rr\sin\alpha}\frac{\partial u_r}{\partial\beta} + \frac{\partial u_\beta}{R\partial r} - \frac{u_\beta}{Rr}. \quad (1.17)$$

Substituting (1.20) into (1.22) for the deformation rate, we have

$$\dot{\varepsilon}_{rr} = \frac{\rho}{RG}\nu[ab\sin 2\alpha\sin(\nu t - \beta) + a^2(\sin\alpha)^2\sin(2\nu t - 2\beta)]\frac{dD(r)}{dr}$$

$$\dot{\varepsilon}_{\alpha\alpha} = \frac{\rho\nu}{GRr}\{ab\sin 2\alpha\sin(\nu t - \beta)[-2C(r) + D(r)] + \frac{a^2}{2}\sin(2\nu t - 2\beta)[C(r)\cos 2\alpha + D(r)(\sin\alpha)^2\}$$

$$\dot{\varepsilon}_{\beta\beta} = \frac{\rho\nu}{GRr}\{ab\sin 2\alpha\sin(\nu t - \beta)[-C(r) + D(r)] + a^2\sin(2\nu t - 2\beta)[-C(r) + [D(r) - C(r)](\sin\alpha)^2]\}$$

$$\dot{\varepsilon}_{r\alpha} = \frac{\rho}{GR}\nu\left[\frac{dC(r)}{dr} + 2\frac{D(r)}{r} - \frac{C(r)}{r}\right][ab\cos 2\alpha\sin(\nu t - \beta) + \frac{a^2}{2}\sin 2\alpha\sin(2\nu t - 2\beta)]$$

$$\dot{\varepsilon}_{\alpha\beta} = -\frac{\rho}{GR}\nu\frac{C(r)}{r}[-ab\sin\alpha\cos(\nu t - \beta) + a^2\cos\alpha\cos(2\nu t - 2\beta)]$$

$$\dot{\varepsilon}_{\beta r} = -2\frac{\rho\nu}{GR}\left[\frac{dC(r)}{dr} + 2\frac{D(r)}{r} - \frac{C(r)}{r}\right][ab\cos\alpha\cos(\nu t - \beta) + a^2\sin\alpha\cos(2\nu t - 2\beta)] \quad (1.18)$$

Substituting (1.18) into (1.16) and performing the necessary calculations, we find the Rayleigh dissipative function

$$\Phi = \frac{8\pi v^2\rho^2R^7F(\mu)}{I5G^2}\frac{L^4(\sin\vartheta)^2}{I_1^2}\left(\frac{(\sin\vartheta)^2}{I_1^2}+\frac{(\cos\vartheta)^2}{I_3^2}\right)$$

$$F(\mu) = \frac{1}{I05(7+5\mu)^2}\left[\frac{15}{4}\lambda^*(2\mu-1)^2 + G^*(767\mu^2+1668\mu+828)\right] \quad (1.19)$$

Dependences (1.17) were used to obtain relation (1.19).

To estimate the coefficients of viscous friction G* and λ* included in (1.19), we assume that the tensors of the coefficients of elasticity and the coefficients of viscosity are similar. With this assumption

$$\lambda^* = \frac{2\mu}{I-2\mu}G^* \quad (1.20)$$

We consider centrally symmetric natural vibrations of a ball in the presence of internal friction. In this case, the radial movement function *u= u(r,t)* satisfies the Lamé equation

$$\frac{2}{R^2}\frac{I-\mu}{I-2\mu}\mathcal{L}[Gu+G^*\acute{u}] - \rho\ddot{u} = 0 \quad (1.21)$$

$$\mathcal{L} = \frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{2}{r^2} \quad (1.22)$$

to boundary condition

$$\sigma_{rr}|_{r=1} = 0 \quad (1.23)$$

The solution of equation (1.22) after separation of variables has the form

$$u(r,t) = \left(\frac{R}{kr}\right)^{\frac{1}{2}} J_{\frac{2}{3}}\left(\frac{kr}{R}\right) exp\,(1\chi t) \quad (1.24)$$

$$k^2 = \frac{R^2(I-2\mu)}{2(I-\mu)}\frac{\rho}{(G+G^*\chi i)} \quad (1.25)$$

Here $J_{\frac{2}{3}}(z) = \left(\frac{2}{\pi z}\right)^{1/2}\left(\frac{\sin z}{z} - \cos z\right)$ - Bessel function with half index.

Expressing the tension σ_{rr} through the movement *u(r,t)* from the boundary condition (1.23) we obtain the transcendental equation for determining the parameter k

$$\mathrm{tg}k = \frac{\kappa}{I-\delta k^2};\ \delta = \frac{(I-\mu)}{2(I-2\mu)}. \quad (1.26)$$

The first positive root of equation (1.31) lies in the interva $1\frac{\pi}{2} < k_1 < \pi$. In particular, at $\mu = 0.3, \delta = 0.875,\ k_1 \approx 2.67$. Let's consider k_1 as known and from the equation (1.26) and find

$$\chi = \chi_1 + \frac{\chi_1^2}{2}\frac{G^*}{G}1;\ \chi_1^2 = \kappa_1^2\left(\frac{2G(I-\mu}{\rho R^2(I-2\mu)}\right), \quad (1.27)$$

where χ_1 - the first natural frequency of elastic vibrations of the rotor. When deriving the formula (1.27), it was taken into account that *G* « G/χ* .

If we denote by η the logarithmic decrement of the damping of the rotor oscillations, then according to (1.32), we have

$$\eta = \frac{\pi G^*}{G}\chi_1$$

Therefore, as an estimate of the coefficient of viscous friction, one can use the relation

$$G^* = \frac{\eta R}{\pi\kappa_1}\left(\frac{\rho G(I-2\mu)}{2(1-\mu)}\right)^{1/2} \quad (1.29)$$

Discussion. The kinetic energy of a dynamically symmetric rigid body moving relative to a fixed point is determined by the expression

$$T = \frac{L^2}{2}\left(\frac{\sin^2\vartheta}{I_1}+\frac{\cos^2\vartheta}{I_3}\right) \quad (1.30)$$

Therefore, bearing in mind that the moment of external forces relative to the center of mass of the gyroscope rotor is zero (L = const) and differentiating the formula (1.34), we obtain the equation for the nutation angle v

$$\dot{\vartheta} = \frac{2I_1I_3T}{(I_3-I_1)L^2\sin 2\vartheta}$$

It is known that the rate of decay of the mechanical energy of the system is equal to the doubled dissipative function Φ, the refore, taking into account (1.5), we come to the following differential equation for the gyroscope nutation angle

$$\frac{\partial\vartheta}{\partial t} = \frac{4\pi(I_3-I_1)G^*\rho^2R^7L^4f(\mu)}{I5I_3^3I_1^3G^2}\sin\vartheta\cos\vartheta\left(\frac{I_3^2\sin^2\vartheta}{I_1^2}+\cos^2\vartheta\right) \quad (1.31)$$

$$f(\mu)=\frac{4}{G^*}F(\mu)=\frac{707\mu^2+2016\mu+1437}{105(7+5\mu)^2} \qquad (1.36)$$

The formula (1.32) was obtained from (1.20) taking into account dependence (1.33).

If we introduce dimensionless time by the formula

$$t=\tau\acute{t}; \quad \tau=\frac{I5I_1^3I_3^3G^2}{4\pi(I_3-I_1)G^*\rho^2R^7L^4f(\mu)} \qquad (1.37)$$

and denote by $s=I_3^2/I_1^2,\ z=tg^2v$, т the obtained equation can be transformed to the form

$$\frac{(I+z)dz}{2z(I+sz)}=-dt$$

Consequently, the secular evolution of the nutation angle will be determined by the equation

$$\mathrm{tg}^{2s}\vartheta(I+3tg^2\vartheta^0)^{s-1}=tg^{2s}\vartheta^0(I+stg^2\vartheta)^{s-1}exp(-2s\acute{t}) \qquad (1.38)$$

Here ϑ^0 initial value of the angle between the axis x_3 of symmetry of the rotor and the axis η3.

Determination of the nutation angle make it possible to determine the speed of alignment of the axis of dynamic symmetry with the kinetic momentum vector, that is, the damping speed, and to draw a conclusion about the time required to prepare the device for operation [8].

For the rotor with an oblate ellipsoid of inertia $I_3 > I_1$ the nutation angle ϑ' decreases over time. Thus, when the rotor moves around the center of mass, the dynamic axis of the rotor symmetry x_3 tends to coincide with the axis η_3, along which the kinetic momentum vector is directed. Formula (1.38) allows one to estimate the axis movement of the rotor symmetry during its spin-up. Substitution of the parameter G^*, which determines the internal friction (1.33), into the time constant (1.37) leads to the following final result

$$\tau=\frac{I5I_1^3I_3^3G^{3/2}R_1}{4\eta(I_3-I_1)\rho^2R^6L^4f(\mu)}\left(\frac{2(I-\mu)}{I-2\mu}\right)^{1/2} \qquad (1.39)$$

Of course, a similar mechanism for damping nutation vibrations also exists for a gyroscope with a non-contact suspension, the rotor of which is a thin spherical shell.

Numerical example 1.1. Let us consider an electrostatic gyroscope with a solid beryllium rotor. The rotor radius R = 0.5 cm, mechanical characteristics: density p = 1850 kg/m^3, shear modulus G = 1,15*1011 Pa, Poisson's ratio μ = 0.3, angular velocity ω = 1.88*104 s-1, I_1 = 0.9*I3, I3 = 0.968*10-8 kgm^2, Kinetic momentum L = 1.824*10-4 kgm^2/s, logarithmic decay rate η = 0.02. In this case, from (1.39) for the time constant we obtain τ = *250 hours.*

Numerical example 1.2. Now we consider an electrostatic gyroscope with a solid aluminum rotor. Density p = 2720 kg/m^3, shear modulus G = 2.65*1010 Pa, Poisson's ratio μ=0.32, the rest of the mechanical characteristics are the same as above. Then from (1.39) we find τ=20 hours.

As can be seen from these numerical results, to maintain the "double rotation" of the rotor, for a sufficiently long time, it is necessary to apply force moments to it, which causes additional errors of the device.

Conclusions. 1. A quantitative estimate of the transient process of the rotor of a non-contact electrostatic gyroscope (ESG), which leads to stationary rotation around the axis of the greatest moment of inertia, is found.

2. The problem of the stress-strain state of the ESG rotor in the presence of inertial forces in a quasi-stationary setting is solved. The deformations of the gyroscope rotor are determined.

3. An expression for the nutation angle as a function of time is obtained, and the time constant of the process of damping of nutation oscillations of the rotor is determined.

4. The drifts are found for a real gyroscope, taking into account the rotor aspherization. The presence of an annular belt gives an error in calculations of no more than 10 percent, therefore, the deformation of the rotor surface caused by the presence of an annular belt in the case of its axisymmetric rotation is insignificant. Therefore, when determining the rotor deformation, the presence of the belt can be neglected.

5. Neglecting the terms containing a small parameter in a degree higher than the first, the expression for the strength function is presented in the form of a series in Legendre polynomials.

6. It was found that when an ellipsoidal body moves in a non-contact suspension, there are at least six equilibrium positions.

7. It has been established that an increase in velocity is acceptable as long as the drift caused by deformation is not comparable to the drift due to manufacturing errors.

8. From the obtained numerical data, it was established that to maintain the "double rotation" of the rotor, for a sufficiently long time, it is necessary to apply force moments to it, which in itself causes additional errors of the device.

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БАЙЛАНЫССЫЗ ГИРОСКОПРОТОРЫНЫҢ ӨТПЕЛІ ПРОЦЕСІН САНДЫҚ БАҒАЛАУ

Аннотация. Мақалада инерцияның ең үлкен моменті осінің айналасында қозғалмайтын айналуға алып келетін байланыссыз гироскоптың роторының өтпелі процесінің сандық бағаларын табуға көңіл бөлінеді. Мәселенің генераторлық шешімі ретінде Эйлер-Пуансо шешімі қолданылады, ал ротордың серпімділігінің әсері, бұзылу әдісі идеяларына сәйкес, Эйлер қозғалысының кішкене толқуы ретінде қарастырылады. Гироскоп роторының деформациясы анықталады. Мақала материалдағы ішкі үйкеліс Кельвин-Фойгт гипотезасына бағынады деп болжайды.

Нутация бұрышы үшін өрнек алынады және ротордың нутационды тербелістерін демпферлеу процесінің уақыт константасы анықталады.

Түйінді сөздер: Жанаспайтын гироскоп, Эйлер бұрыштары, инерция үштері, Лежандр көпмүшелігі, Пуассон коэффициенті, диссипативті функция, ішкі үйкеліс, гироскоптың нутация бұрышының теңдеуі.

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КОЛИЧЕСТВЕННЫЕ ОЦЕНКИ ПЕРЕХОДНОГО ПРОЦЕССА РОТОРА НЕКОНТАКТНОГО ГИРОСКОПА

Аннотация. В статье основное внимание уделяется нахождению количественных оценок переходного процесса ротора неконтактного гироскопа, приводящего к стационарному вращению вокруг оси наибольшего момента инерции. Решение Эйлера – Пуансо применяется в качестве порождающего решение задачи, а влияние упругости ротора в соответствии с идеями метода возмущений рассматривается как малое возмущение эйлерова движения.

Определены деформации ротора гироскопа. В статье предполагается, что внутреннее трение в материале подчиняется гипотезе Кельвина-Фойгта.

Получено выражение угла нутации в зависимости от времени и определена постоянная времени процесса затухания нутационных колебаний ротора.

Ключевые слова: неконтактный гироскоп, углы Эйлера, силы инерций, полином Лежандра, коэффициент Пуассона, диссипативная функция, внутреннее трение, уравнение угла нутации гироскопа.

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